Statistical Theory of Extremes
Homepage: http://www.gathacognition.com/book/gcb14
http://dx.doi.org/10.21523/gcb1

Part 1
Probabilistic Patterns of Univariate Statistical Extremes

Annex 1

On The "Duality" between Extremes and Sums

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Editor(s)	**Published Online**
J.C. Tiago de Oliveira	23 June 2017

For simplicity we will deal with some "duality" between sums (or averages) and maxima, the translation to minima being obvious from the relation $\min_1^n \{X_i\} = -\max_1^n \{-X_i\}$.

The "duality" is expressed by the two columns in correspondence, where there are various gaps. $F(.), F(.,.), \ldots$ and $\varphi(.), \varphi(.,.), \ldots$ will denote the distribution functions and the characteristic functions.

Originally published in 'Statistical Analysis of Extremes', 1997, 2016
http://dx.doi.org/10.21523/gcb1.1705

Sums	Maxima
$S_k = \sum_1^k X_i$	$M_k = \max_1^k \{X_i\}$
$\varphi x(t) = M_X(e^{itX})$: ch. f. of X	$F_X(t) = \text{Prob}\{X \leq x\}$: d. f. of X
X_i indep.: $\varphi_{S_k}(t) = \prod_1^k \varphi_i(t)$	X_i indep.: $F_{M_k}(x) = \prod_1^n F_i(x)$
X_i i.i.d.: $\varphi_{S_k}(t) = \varphi^k(t)$ (X, Y) indep: $\varphi aX + bY(t)$ $= \varphi X(a\,t)\, \varphi Y(b\,t)$	X_i i.i.d.: $F_{M_k}(x) = F^k(x)$ (X, Y) indep: $F_{\max(X+a, Y+b)}$ $= F_X(x-a) F_Y(x-b)$
$(+\,,.)$	$(\max\,, +)$
$M(aX + b) = a\,M(X) + b$	…
$V(aX + b) = a^2\,V(X)$	…
If $\{X_i\}$ i.i.d. have μ, σ^2 then $\varphi_{(s_k - k\mu)/\sqrt{k}\,\sigma}(t) \to (e^{-t^2/2})$ (Central Limit Theorem); in the general case "sometimes" the ch. f. of the normal law may be substituted by that of an indefinitely divisible law ; $\mu = \varphi'_X(0)/i\,, \sigma^2 = \varphi'_X(0)^2 - \varphi''_X(0)$, in the usual case.	If $\{X_i\}$ are i.i.d. there "sometimes" exist $(\lambda_k, \delta_k > 0)$ such that $\text{Prob}\{(M_k - \lambda_n)/\delta_n \leq x\} = F^k(\lambda_k + \delta_k\,x) \to \tilde{L}(x), \tilde{L}(x)$ then being $\Psi_\alpha(x)\,, \Lambda(x)$ or $\Phi_\alpha(x)$; for $\Lambda(x)$ we have $n(1 - F(\lambda_k)) \to 1, k(1 - F(\lambda_k + \delta_k)) \to e^{-1}$ or $\delta_n \sim 1/n\,F'(\lambda_n)$; there are corresponding results for Φ_α and Φ_α.
If $(X\,, Y)$ has a binormal distribution standard normal margins and correlation coefficient then $Z = aX + bY$ has a normal distribution $N(x/\sigma(a, b))$ with $\sigma(a, b) = a^2 + b^2 + 2\,\rho\,(a\,b)$.	If $(X\,, Y)$ has a bivariate distribution with reduced Gumbel margins then $Z = \max(X - a\,, Y - b)$ has a Gumbel distribution $\Lambda(z - \lambda(a\,, b))$ with $\lambda(a\,, b) = \log\{(e^{-a} + e^{-b})k(b - a)\}$
If $\rho = 0$ (independence), $\sigma(a, b) = 1$ iff $a^2 + b^2 = 1$; in the case $C(X\,, Z) = a$.	If $k(w) = 1$ (independence), $\lambda(a\,, b) = 0$ if $e^{-a} + e^{-b} = 1$; in that case $\text{Prob}\{Z \leq X - a\} = \text{Prob}\{X - b \leq X - a\} = e^{-a}$.

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