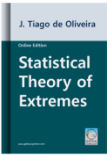




# Statistical Theory of Extremes

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## Part 1

Probabilistic Patterns of Univariate Statistical Extremes

### Annex 1

## On The “Duality” between Extremes and Sums

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For simplicity we will deal with some “duality” between sums (or averages) and maxima, the translation to minima being obvious from the relation

$$\min_{1 \leq i \leq n} \{X_i\} = -\max_{1 \leq i \leq n} \{-X_i\}.$$

The “duality” is expressed by the two columns in correspondence, where there are various gaps.  $F(\cdot)$ ,  $F(\cdot, \cdot)$ , ... and  $\varphi(\cdot)$ ,  $\varphi(\cdot, \cdot)$ , ... will denote the distribution functions and the characteristic functions.

Sums	Maxima
$S_k = \sum_{i=1}^k X_i$ $\varphi_X(t) = M_X(e^{itX}): \text{ch. f. of } X$ $X_i \text{ indep.: } \varphi_{S_k}(t) = \prod_{i=1}^k \varphi_i(t)$ $X_i \text{ i. i. d.: } \varphi_{S_k}(t) = \varphi^k(t)$ $(X, Y) \text{ indep.: } \varphi_{aX+bY}(t) = \varphi_X(at) \varphi_Y(bt)$ $(+, \cdot)$ $M(aX+b) = a M(X) + b$ $V(aX+b) = a^2 V(X)$ <p>If <math>\{X_i\}</math> i.i.d. have <math>\mu, \sigma^2</math> then</p> $\frac{\varphi(t) - e^{it\mu}}{(s_k - k\mu)/\sqrt{k}\sigma} \rightarrow (e^{-t^2/2})$ <p>(Central Limit Theorem); in the general case “sometimes” the ch. f. of the normal law may be substituted by that of an indefinitely divisible law ; <math>\mu = \varphi'_X(0)/i, \sigma^2 = \varphi'_X(0)^2 - \varphi''_X(0)</math>, in the usual case.</p> <p>If <math>(X, Y)</math> has a binormal distribution standard normal margins and correlation coefficient then <math>Z = aX + bY</math> has a normal distribution <math>N(x/\sigma(a, b))</math> with <math>\sigma(a, b) = a^2 + b^2 + 2\rho(a, b)</math>.</p> <p>If <math>\rho = 0</math> (independence), <math>\sigma(a, b) = 1</math> iff <math>a^2 + b^2 = 1</math>; in the case <math>C(X, Z) = a</math>.</p>	$M_k = \max_{1 \leq i \leq k} \{X_i\}$ $F_X(t) = \text{Prob}\{X \leq x\}: \text{d. f. of } X$ $X_i \text{ indep.: } F_{M_k}(x) = \prod_{i=1}^n F_i(x)$ $X_i \text{ i. i. d.: } F_{M_k}(x) = F^k(x)$ $(X, Y) \text{ indep.: } F_{\max(X+a, Y+b)} = F_X(x-a)F_Y(y-b)$ $(\max, +)$ <p>...</p> <p>...</p> <p>If <math>\{X_i\}</math> are i.i.d. there “sometimes” exist <math>(\lambda_k, \delta_k &gt; 0)</math> such that <math>\text{Prob}\{(M_k - \lambda_n)/\delta_n \leq x\} = F^k(\lambda_k + \delta_k x) \rightarrow \tilde{L}(x), \tilde{L}(x)</math> then being <math>\Psi_\alpha(x), \Lambda(x)</math> or <math>\Phi_\alpha(x)</math>; for <math>\Lambda(x)</math> we have <math>n(1 - F(\lambda_k)) \rightarrow 1, k(1 - F(\lambda_k + \delta_k)) \rightarrow e^{-1}</math> or <math>\delta_n \sim 1/n F'(\lambda_n)</math>; there are corresponding results for <math>\Phi_\alpha</math> and <math>\Phi_\alpha</math>.</p> <p>If <math>(X, Y)</math> has a bivariate distribution with reduced Gumbel margins then <math>Z = \max(X - a, Y - b)</math> has a Gumbel distribution <math>\Lambda(z - \lambda(a, b))</math> with <math>\lambda(a, b) = \log\{(e^{-a} + e^{-b})k(b - a)\}</math></p> <p>If <math>k(w) = 1</math> (independence), <math>\lambda(a, b) = 0</math> if <math>e^{-a} + e^{-b} = 1</math>; in that case <math>\text{Prob}\{Z \leq X - a\} = \text{Prob}\{X - b \leq X - a\} = e^{-a}</math>.</p>

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